DEVELOPMENT OF NONSTEADY FREE CONVECTION UNDER THE ACTION OF MOVING VERTICAL PLATES

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The effect of free convection on the tangential stress of a vertical heated plate when the plate is suddenly introduced into the motion and its temperature is changed is considered.

In [1-4] the nonsteady free convection around an infinite vertical motionless heated plate was investigated, for different initial and boundary conditions. In [1, 2] the problem was solved on the basis of similarity transformations, and in [3, 4] a Laplace transformation was used. In [5] the problem of free-convection oscillatory motion about a vertical uniformly moving plate was considered.

Below, the free convection about a vertical heated plate is investigated in the case when the plate velocity and temperature change.

Consider the motion of a viscous incompressible liquid in the semi-infinite region $y > 0$ bounded by a rigid vertical wall (at $y = 0$), which begins to move in its plane at a velocity $u_0F(\tau)$ at time $t = 0$. The plate temperature changes suddenly from T_0 to $T_0 + T_\omega G(\tau)$. Since the fluid motion is due to the parallel displacement of the plate, the initial system of equations is written in the form

$$\frac{\partial u}{\partial t} = g\beta(T - T_0) + \nu\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{2}$$

Introducing the dimensionless variables

$$\eta = yu_0/\nu, \quad u_1 = u/u_0, \quad \tau = u_0^2 t/\nu, \quad \theta = (T - T_0)/T_\omega \tag{3}$$

Eqs. (1) and (2) may be rewritten in the form

$$\frac{\partial u_1}{\partial \tau} = \mathrm{Gr}\theta + \frac{\partial^2 u_1}{\partial \eta^2}, \tag{4}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\sigma}\frac{\partial^2 \theta}{\partial \eta^2}. \tag{5}$$

The initial and boundary conditions are taken to be

$$u_1 = \theta = 0 \quad \text{for} \quad \tau \leqslant 0,$$

$$u_1 = F(\tau), \quad \theta = G(\tau) \quad \text{for} \quad \eta = 0, \ \tau > 0, \tag{6}$$

$$u_1 \to 0, \quad \theta \to 0 \quad \text{for} \quad \eta \to \infty, \ \tau > 0. \tag{7}$$

Applying a Laplace transformation to Eqs. (4) and (5) and using Eq. (6), the result obtained is

$$\frac{d^2 u_1^*}{d\eta^2} - pu_1^* = -\mathrm{Gr}\theta^*, \tag{8}$$

$$\frac{d^2\theta^*}{d\eta^2} = \sigma p\theta^*, \tag{9}$$

where

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$$u_1^* = \int_0^\infty u_1(\eta, \tau)\exp(-p\tau)\,d\tau \text{ and } \theta^* = \int_0^\infty \theta(\eta, \tau)\exp(-p\tau)\,d\tau \quad (p > 0). \tag{10}$$

The boundary conditions in Eq. (7) for the function in Eq. (10) are transformed as follows

$$\begin{aligned} &u_1^* = F^*, \quad \theta^* = G^* \quad \text{for} \quad \eta = 0, \\ &u_1^* \to 0, \quad \theta^* \to 0 \quad \text{for} \quad \eta \to \infty. \end{aligned} \tag{11}$$

The solution of Eqs. (9) and (8) with the boundary conditions in Eq. (11) takes the form

$$\theta^* = G^* \exp(-\sqrt{p\sigma}\,\eta), \tag{12}$$

$$u_1 = F^* \exp(-\sqrt{p}\eta) + \frac{\mathrm{Gr}\,G^*}{p(\sigma-1)}[\exp(-\sqrt{p}\eta) - \exp(-\sqrt{p\sigma}\eta)] \quad (\sigma \neq 1), \tag{13}$$

$$u_1 = F^* \exp(-\sqrt{p}\eta) - \frac{\eta\,\mathrm{Gr}\,G^*}{2\sqrt{p}}\exp(-\sqrt{p}\eta) \quad (\sigma = 1). \tag{14}$$

Applying the inverse transformation to Eqs. (12)-(14) gives

$$\theta = \frac{\sqrt{\sigma}\,\eta}{2\sqrt{\pi}}\int_0^\infty G(\tau-\lambda)\lambda^{-3/2}\exp(-\sigma\eta^2/4\lambda)\,d\lambda, \tag{15}$$

$$\begin{aligned} u_1 = {} & \frac{\eta}{2\sqrt{\pi}}\int_0^\infty F(\tau-\lambda)\lambda^{-3/2}\exp(-\eta^2/4\lambda)\,d\lambda + \frac{\mathrm{Gr}}{\sigma-1}\int_0^\infty G(\tau-\lambda)\times \\ & \times[\operatorname{erfc}(\eta/2\sqrt{\lambda}) - \operatorname{erfc}(\sqrt{\sigma}\eta/2\sqrt{\lambda})]\,d\lambda \quad (\sigma\neq 1), \end{aligned} \tag{16}$$

$$u_1 = \frac{\eta}{2\sqrt{\pi}}\int_0^\infty F(\tau-\lambda)\lambda^{-3/2}\exp(-\eta^2/4\lambda)\,d\lambda + \frac{\mathrm{Gr}\,\eta}{2\sqrt{\pi}}\int_0^\infty G(\tau-\lambda)\lambda^{-1/2}\exp(-\eta^2/4\lambda)\,d\lambda \quad (\sigma = 1). \tag{17}$$

The expressions for θ and u_1 obtained may be used to analyze various cases of initial conditions.

Example 1

The plate velocity and temperature change in a steplike manner. In this case, $F(\tau) = G(\tau) = H(\tau)$, where

$$H(\tau) = \begin{cases} 0, & \tau < 0, \\ 1, & \tau \geqslant 0. \end{cases} \tag{18}$$

Substituting these expressions for $F(\tau)$ and $G(\tau)$ into Eqs. (15)-(17) gives [6]

$$\theta = H(\tau)\, i^0 \operatorname{erfc}(\sqrt{\sigma}\xi), \tag{19}$$

$$u_1 = H(\tau)\left[i^0\operatorname{erfc}(\xi) - \frac{4\,\mathrm{Gr}\,\tau}{\sigma-1}\left\{i^2\operatorname{erfc}(\xi) - i^2\operatorname{erfc}(\sqrt{\sigma}\xi)\right\}\right] (\sigma \neq 1), \tag{20}$$

$$u_1 = H(\tau)\,[i^0\operatorname{erfc}(\xi) + 2\tau\,\mathrm{Gr}\,\xi\, i\operatorname{erfc}(\xi)]\,(\sigma = 1), \tag{21}$$

where

$$\begin{aligned} &\xi = \eta/2\sqrt{\tau}, \quad i^0\operatorname{erfc}(z) = \operatorname{erfc}(z), \\ &i^n\operatorname{erfc}(z) = \int_z^\infty i^{n-1}\operatorname{erfc}(t)\,dt \quad (n = 1, 2, 3, \ldots). \end{aligned} \tag{22}$$

The tangential stress at the plate is determined by the expression

$$\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = \frac{H(\tau)}{\sqrt{\pi\tau}}\left[-1 + \frac{2\tau\mathrm{Gr}}{\sqrt{\sigma}+1}\right] \tag{23}$$

for all σ. Hence it is evident that the tangential stress at the plate increases with rise in τ or Gr, and falls with increase in the Prandtl number σ. It is interesting to note that the tangential stress at the wall vanishes at $\tau = \tau_1$, where

$$\tau_1 = \sqrt{\sigma} + 1/2\,\mathrm{Gr} \tag{24}$$

for all σ. It is clear from Eq. (24) that the time of flux breakaway after the beginning of the motion decreases with rise in Gr and increases with rise in the Prandtl number σ.

Example 2

The plate begins to move in a steplike manner, and its temperature increases instantaneously. Here $F(\tau) = H(\tau)$ and $G(\tau) = \tau H(\tau)$. For these values of $F(\tau)$ and $G(\tau)$, Eqs. (15)-(17) give

$$\theta = H(\tau)\,[i^0 \operatorname{erfc}(\sqrt{\sigma}\xi) - 2\tau\sqrt{\sigma}\xi i \operatorname{erfc}(\sqrt{\sigma}\xi)], \tag{25}$$

$$u_1 = H(\tau)\left[i^0 \operatorname{erfc}(\xi) \frac{16\mathrm{Gr}\,\tau^2}{\sigma - 1}\left\{i^4 \operatorname{erfc}(\xi) - i^4 \operatorname{erfc}(\sqrt{\sigma}\xi)\right\}\right] \quad (\sigma \neq 1), \tag{26}$$

$$u_1 = H(\tau)\,[i^0 \operatorname{erfc}(\xi) + 8\mathrm{Gr}\,\tau^2\xi i^3 \operatorname{erfc}(\xi)] \quad (\sigma = 1). \tag{27}$$

The tangential stress at the plate is determined by the expression

$$\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = \frac{H(\tau)}{\sqrt{\pi\tau}}\left[-1 + \frac{4\tau^2\mathrm{Gr}}{3(\sqrt{\sigma}+1)}\right] \tag{28}$$

for all σ. In this case, the time τ_2 at which the tangential stress vanishes at the plate is defined by the relation

$$\tau_2 = \left(\frac{3}{2}\tau_1\right)^{1/2} \tag{29}$$

for all σ, where τ_1 is found from Eq. (25).

Example 3

The plate begins to move with instantaneous acceleration, while its temperature changes in a steplike manner. Here $F(\tau) = \tau H(\tau)$, $G(\tau) = H(\tau)$. In this case, θ and u_1 are determined as

$$\theta = H(\tau)\, i^0 \operatorname{erfc}(\sqrt{\sigma}\xi), \tag{30}$$

$$u_1 = \tau H(\tau)\,[i^0 \operatorname{erfc}(\xi) - 2\xi i \operatorname{erfc}(\xi) + \frac{4\mathrm{Gr}}{\sigma - 1}\{i^2 \operatorname{erfc}(\xi) - i^2 \operatorname{erfc}(\sqrt{\sigma}\xi)\}] \quad (\sigma \neq 1), \tag{31}$$

$$u_1 = \tau H(\tau)\,[i^0 \operatorname{erfc}(\xi) + 2(\mathrm{Gr} - 1)\xi i \operatorname{erfc}(\xi)] \quad (\sigma = 1). \tag{32}$$

The tangential stress at the plate is determined in this case as

$$\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = 2H(\tau)\left(\frac{\tau}{\pi}\right)^{1/2}\left[-1 + \frac{\mathrm{Gr}}{\sqrt{\sigma}+1}\right] \tag{33}$$

for all σ. It is interesting to note that the tangential stress at the plate vanishes when

$$\mathrm{Gr} = \sqrt{\sigma} + 1. \tag{34}$$

In this case the flux breaks away immediately after the plate begins to move.

Example 4

The plate begins to move with instantaneous acceleration, while its temperature rises instantaneously. Here $F(\tau) = \tau H(\tau)$, $G(\tau) = \tau H(\tau)$, and Eqs. (15)-(17) give the result

$$\theta = \tau H(\tau)\,[i^0 \operatorname{erfc}(\sqrt{\sigma}\xi) - 2\sqrt{\sigma}\xi i \operatorname{erfc}(\sqrt{\sigma}\xi)], \tag{35}$$

$$u_1 = \tau H(\tau)\left[i^0 \operatorname{erfc}(\xi) - 2\xi i \operatorname{erfc}(\xi) - \frac{16\mathrm{Gr}\,\tau}{\sigma - 1}\{i^4 \operatorname{erfc}(\xi) - i^4 \operatorname{erfc}(\sqrt{\sigma}\xi)\}\right] \quad (\sigma \neq 1), \tag{36}$$

$$u_1 = \tau H(\tau)\,[i^0 \operatorname{erfc}(\xi) - 2\xi i \operatorname{erfc}(\xi) + 8\mathrm{Gr}\,\tau\xi i^3 \operatorname{erfc}(\xi)] \quad (\sigma = 1). \tag{37}$$

The tangential stress at the plate is

$$\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = 2H(\tau)\left(\frac{\tau}{\pi}\right)^{1/2}\left[-1 + \frac{2\mathrm{Gr}\,\tau}{3(\sqrt{\sigma}+1)}\right] \tag{38}$$

for all σ. In this case, the tangential stress at the plate vanishes for a time τ_4 given by

$$\tau_4 = \frac{4}{3}\tau_1 \tag{39}$$

for all σ, with τ_1 as in Eq. (24).

It is evident from Eqs. (23), (29), (33), and (38) that the tangential stress at the plate increases with rise in τ and Gr and decreases with rise in σ. It is also seen that in the first, second, and fourth cases the flux breaks away after a certain time which depends on the Prandtl and Grashof numbers.

NOTATION

Gr, Grashof number $g\beta T_\omega \nu/u_0^3$; g, acceleration due to gravity; T, fluid temperature; T_0, plate temperature for $t < 0$; T_ω, change in plate temperature for $t = 0$; t, time; u, fluid velocity in the x direction; u_0, change in plate velocity for $t = 0$; u_1, dimensionless velocity (u/u_0); y, normal coordinate; α, thermal conductivity; β, thermal expansion coefficient; η, dimensionless coordinate (yu_0/ν); θ, dimensionless temperature $(T - T_0)/T_\omega$; ν, kinematic viscosity; σ, Prandtl number (ν/α); τ, dimensionless time $(u_0^2 t/\nu)$.

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CALCULATION OF THE HEATING OF POLYDISPERSE PARTICLES IN A GAS

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The problem of the heating of polydisperse particles in a gas is solved with allowance for the temperature field inside a particle and the variation of the gas temperature.

At the time $t = 0$ let an adiabatically closed volume of gas with a temperature $T(0)$ be uniformly filled with homogeneous, polydisperse, spherical particles having a temperature T_0. The problem consists in determining the average temperatures of the particles and the gas at any time. The energy equation is written in the form

$$c\rho \frac{dT}{dt} + 4\pi c_p \rho_p h_0 \int_0^\infty f(r_1) \left[\int_0^{r_1} r^2 \frac{\partial T_p}{\partial t} dr \right] dr_1 = 0. \quad (1)$$

The temperature of the particles is determined from the heat-conduction equation

$$\frac{\partial T_p}{\partial t} = a \nabla^2 T_p. \quad (2)$$

We choose the initial temperature of the particles as the origin of the temperature frame, and then the initial and boundary conditions take the form

$$T_p(t = 0) = 0; \quad \left. \frac{\partial T_p}{\partial r} \right|_{r=0} = 0;$$

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